Cost Minimization of a WWTP Using an Augmented Lagrangian Pattern Search Based Solver

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Abstract This paper describes a derivative-free method that aims to solve an optimization problem arising from the mathematical formulation of an activated sludge system in which the objective is to minimize the investment and operation costs. The method relies on an augmented Lagrangian technique that uses a pattern search for solving bound constrained subproblems.

Keywords Activated Sludge System, Secondary Settler, Pattern Search Method, Augmented Lagrangian.

Introduction

With the growing need to decrease the installation and operation costs of a wastewater treatment plant (WWTP), the search for a minimum cost design is becoming more and more challenging. To be able to achieve an optimal WWTP design an optimization procedure that considers the mathematical modelling of the secondary treatment, an activated sludge system that consists of an aeration tank and a secondary settler, and the definition of a cost function is carried out. The impact of the primary treatment on the cost of the secondary treatment is also reported.

In the presence of non-smooth functions as encountered in some of the mathematical equations involved in the model, a derivative-free optimization technique is the most appropriate. So, a pattern search algorithm is proposed. This algorithm relies on an augmented Lagrangian function in order to obtain a solution that satisfies the equality and inequality constraints of the problem.

Methods

To model the aeration tank, where the biological reactions take place, the ASM1 model (Henze *et al.*, 1986) is used. The tank is considered a CSTR in steady state. The balances around this unit define some of the constraints of the mathematical model. We refer to Espírito Santo *et al.*, (2006a) for more details. To model the secondary settler, which plays a crucial role in the wastewater treatment, a combination of the two traditional models ATV (Ekama *et al.*, 1978) and the double exponential (DE) (Takács *et al*., 1991) is used. Previous work (Espírito Santo *et al.,* 2006b) shows that this combined model is prepared to overcome PWWF events without over dimensioning and provided the most equilibrated WWTP design when compared with the other two used separately. When these three designs were introduced in the GPS-X simulator (http://www.hydromantis.com) and a stress condition of a PWWF value of 5 times the normal flow was imposed, only the combined model was able to support this adverse condition maintaining the quality of the effluent under the values imposed by the portuguese law.

The system behaviour, in terms of concentration and flows, may be predicted by balances. They were done to the suspended matter, dissolved matter and flows. Another important group of constraints in the mathematical model is a set of linear equalities that define composite variables as some state variables are, most of the time, not available for evaluation. Some system variables definitions were also added to the model in order to define the system correctly. All the variables in the model must be nonnegative, although more restricted bounds are imposed to some of them due to operational consistencies. Finally, the quality of the effluent has to be imposed. The quality constraints are usually derived from law restrictions. The most used are related with limits in the *COD*, *N* and *TSS* at the effluent.

A cost function is used to describe the installation and operation costs of a WWTP in a way that reflects the behaviour of each unitary process. In the present study, only the aeration tank and the secondary settler are considered. Based on the work done by Tyteca *et al.* (1977) the basic structure $C = aZ^b$, where C represents the cost and Z the variable that most influences the design of the unitary process, is used. The parameters a and b are estimated using a least squares technique considering real data collected from a portuguese WWTP company. The total cost (TC) function is the sum of the operation and investment costs, in a present value basis, and is given by

$$TC = 174.2V_a^{1.07} + 12487G_S^{0.62} + 114.8G_S + 955.5A_s^{0.97} + 41.3(A_s h)^{1.07} \tag{1}$$

where V_a is the aeration tank volume, G_S is the air flow, A_s and h are the secondary settler surface area and depth, respectively. The obtained mathematical model considers the objective function (1), has 64 parameters, 115 decision variables and 105 constraints, where 67 are nonlinear equalities, 37 are linear equalities and there is only 1 nonlinear inequality. Eleven variables are bounded below and above and the remaining are bounded below. For a more thorough analysis of all the equations the reader is referred to Espírito Santo *et al.* (2006a).

The formulation of the mathematical model incorporates some constraints, from the DE model, that are non-smooth functions. Optimization methods that rely on derivative information should not be used. Thus, a derivative-free algorithm based on a pattern search paradigm is proposed. For simplicity, consider the mathematical formulation of the problem as follows

$$\underset{x \in \Psi \subset \Re^n}{\text{minimize}} \ f(x) \tag{2}$$

where x contains the n decision variables of the problem, $\Psi = \{x : b(x) = 0, g(x) \le 0, l \le x \le u\}$ denotes the feasible region, f is the objective function (in our case, is the total cost function), $b(x) = 0$ contains the (m) equality constraints and $g(x) \le 0$ the (p) inequality constraints. To solve problem (2), a penalty multiplier technique is used. We refer to Bertsekas (1996) for details. This type of technique solves a sequence of very simple subproblems with objective functions that aim to penalize the constraints violation, known as penalty functions. The following augmented Lagrangian penalty function

$$\Phi(x;\lambda,\delta,\mu) = f(x) + \sum_{i=1}^{m} \lambda_i b_i(x) + \frac{1}{2\mu}\sum_{i=1}^{m} b_i(x)^2 + \frac{\mu}{2}\sum_{i=1}^{p}\left(\max\left(0,\delta_i + \frac{g_i(x)}{\mu}\right)^2 - \delta_i^2\right)$$

is considered, where μ is a positive penalty parameter, λ and δ are the Lagrange multipliers vectors, associated with the equality and inequality constraints respectively. Note that the function $\Phi(x;.)$ only incorporates the equality and inequality constraints, leaving out the simple bounds $l \le x \le u$. Thus, the corresponding subproblems are defined as

$$\underset{x\in\Omega\subset\Re^n}{\text{minimize}} \quad \Phi(x;\lambda^j,\delta^j,\mu^j) \tag{3}$$

where the feasible region is $\Omega = \{x : l \le x \le u\}$. The solution of (3), for each set of fixed λ^j, δ^j and μ^j, gives an approximation to the solution of (2). Here, the index j is the iteration counter. As $j \to \infty$ and $\mu^j \to 0$, the solutions of subproblems (3) converge to the solution of (2). The Lagrange multipliers (λ^j,δ^j) are estimated in this iterative process according to appropriate updating formulae. This outer iterative process is halted when an approximation that satisfies the first-order optimality conditions for problem (2) is found.

Algorithm 1: (augmented Lagrangian multiplier method)
Initialize variables and algorithm parameters
while convergence criteria are not satisfied **do**
  Inner iterative process: approximately solve subproblem (3)
  Update multipliers δ^j and λ^j, penalty parameter μ^j, $j \leftarrow j+1$
end while

Each subproblem (3) is solved by a pattern search method (Lewis *et al.*, 1999) that does not require any derivative information. In this method a series of exploratory moves about a current iterate, x^k, is conducted in order to find a new iterate, $x^{k+1} = x^k + \Delta^k s^k$, with a lower objective function value, where Δ^k represents the step length and s^k determines the direction of the step. The step $\Delta^k s^k$ is computed by the Hooke and Jeeves (HJ) exploratory moves (Hooke *et al.*, 1961). The index k is the iteration counter in this inner iterative process.

Algorithm 2: (pattern search method)
Initialize with $x^k \in \Omega$
while termination criterion is not satisfied **do**
  Compute $\Delta^k s^k$ using HJ exploratory moves such that $x^k + \Delta^k s^k \in \Omega$
  if $\Phi(x^k;.) - \Phi(x^k + \Delta^k s^k;.) > 0$ **then** $x^{k+1} = x^k + \Delta^k s^k$ **else** $x^{k+1} = x^k$ **end if**
  Update Δ^k and s^k, $k \leftarrow k+1$
end while

For a more thorough analysis of the proposed method the reader is referred to Espírito Santo (2007).

Results and discussion

The following results were obtained using the data provided by a WWTP company builder, corresponding to four small towns in the interior north of Portugal – Alijó, Murça, Sabrosa and Sanfins do Douro. Two types of experiments were carried out. One does not consider a primary treatment and the other has a 40% efficiency primary treatment.

Table 1 shows the optimal values for some of the decision variables, namely, the aeration tank volume, the air flow, the area and depth of the secondary settler, and *COD*, *TSS* and *N* at the effluent and the total cost (in millions of euros).

Table 1 Results for the four WWTPs.

	efficiency	V_a	G_S	A_s	h	COD	TSS	N	total cost
Alijó	0	2632	108	654	23.8	41.1	22.8	6.4	2.79
	40	1738	100	811	16.4	44.6	11.2	6.5	2.41
Murça	0	1599	102	746	16.5	12.8	10.4	3.1	2.24
	40	913	101	371	16.9	74.7	24.2	9.5	1.25
Sabrosa	0	1344	100	707	17.5	88.7	20.1	9.1	2.14
	40	914	133	557	9.6	28.7	14.6	9.7	1.36
Sanfins do Douro	0	1263	103	703	14.0	51.7	24.0	10.5	1.9
	40	1179	158	404	10.7	51.2	16.3	9.7	1.28

The presence of the primary treatment causes a reduction in the total cost: a slight reduction (13.6%) in the largest WWTP (Alijó) and a more significant reduction in the other plants (from 32.6% to 44.2%). In all cases, the *COD*, *TSS* and *N* law limits (125, 35 and 15, respectively) were never achieved (see columns 7, 8 and 9 in Table 1), meaning that the obtained solutions are robust. The volume is reduced in the presence of the primary treatment but the air flow is maintained at the same range of values. The presence of the primary treatment also causes a reduction in the settling area and in the settler tank depth, except for the Alijó plant in the first case, and Murça plant in the second case.

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